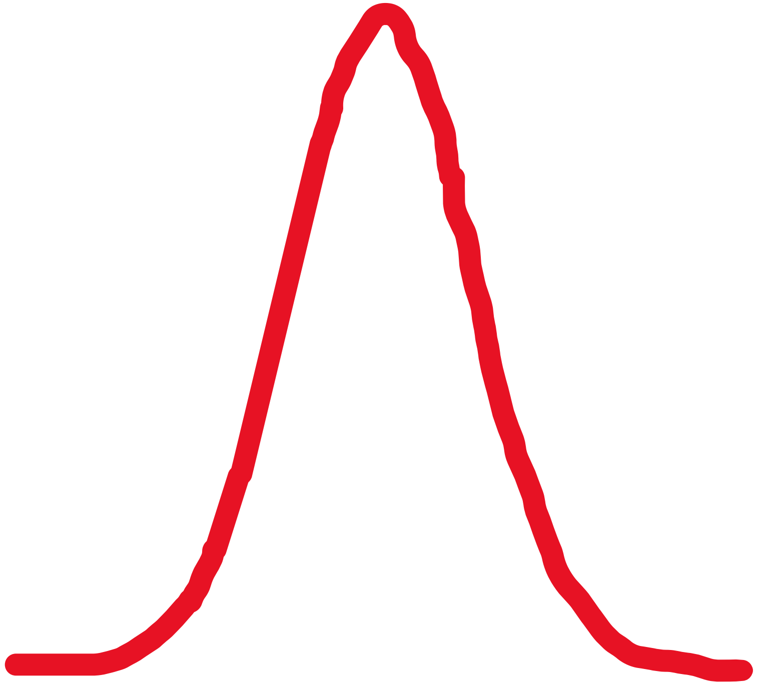
17 – Confidence Intervals (unknown σ)

Reference: [ES] 6.2

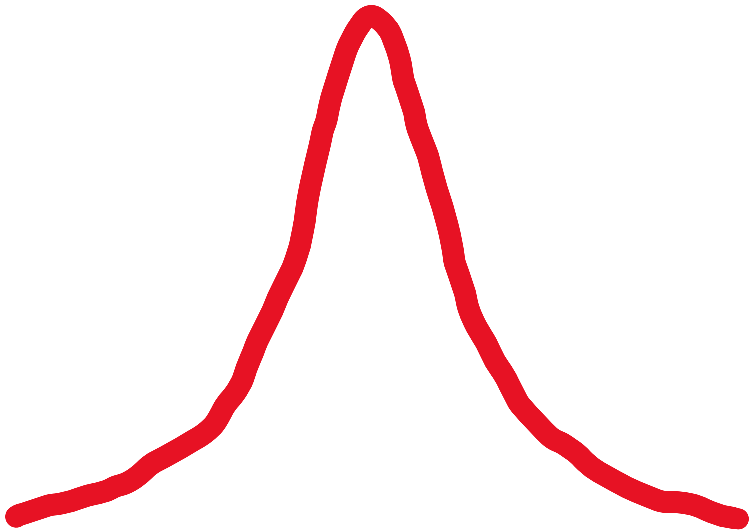
Sampling Distribution

Given pop. distribution with mean μ,
the sampling distribution of size n looks like:

For σ known:



For σ unknown:
Use sample mean s_x to approximate σ.



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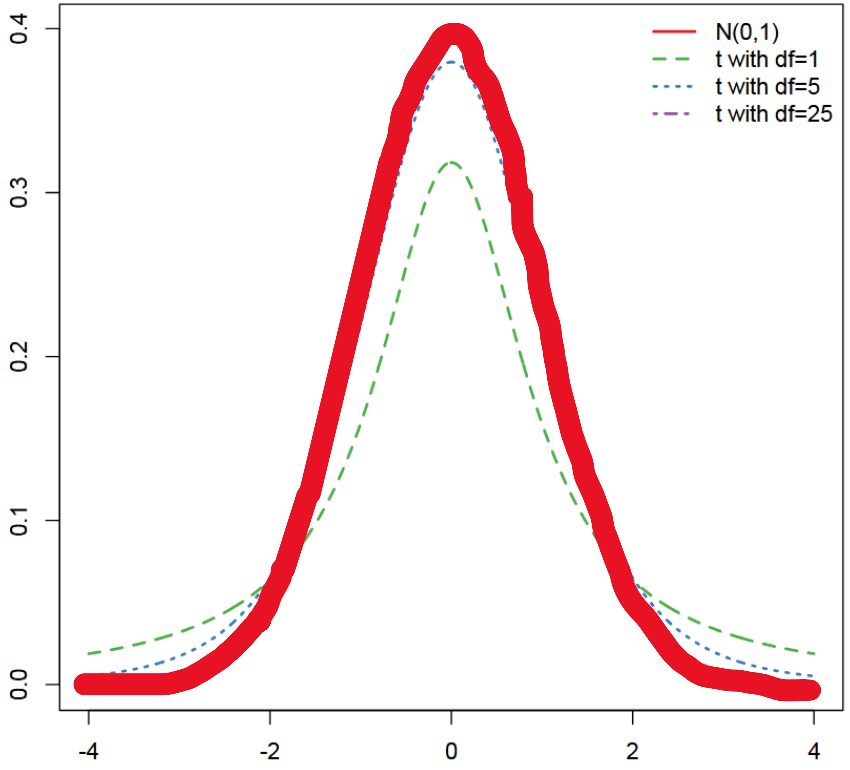

Sampling distribution of $\bar{x}$ is normal with $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.

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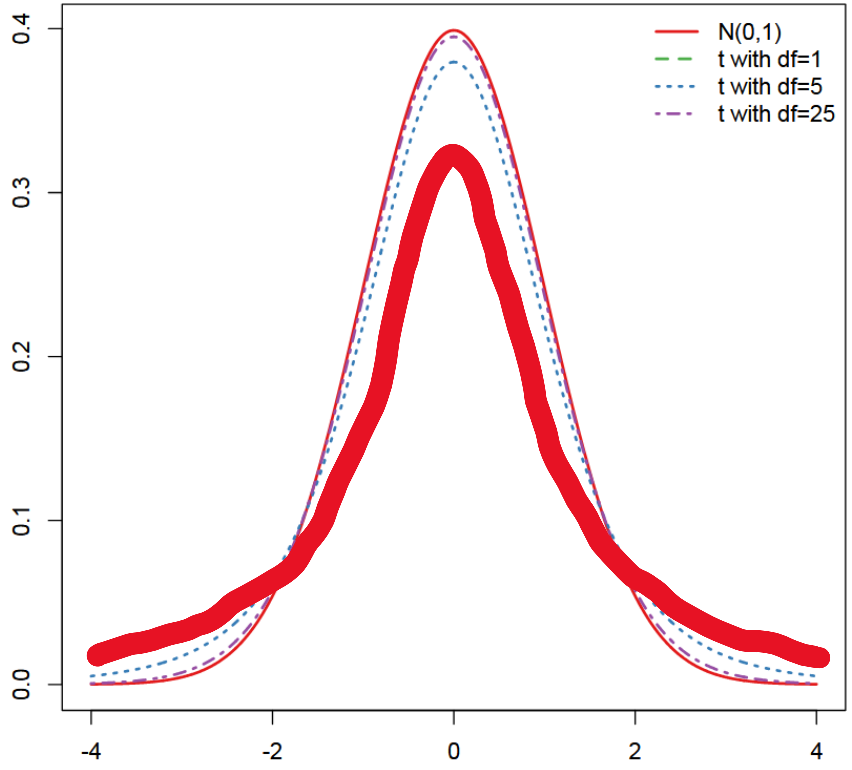

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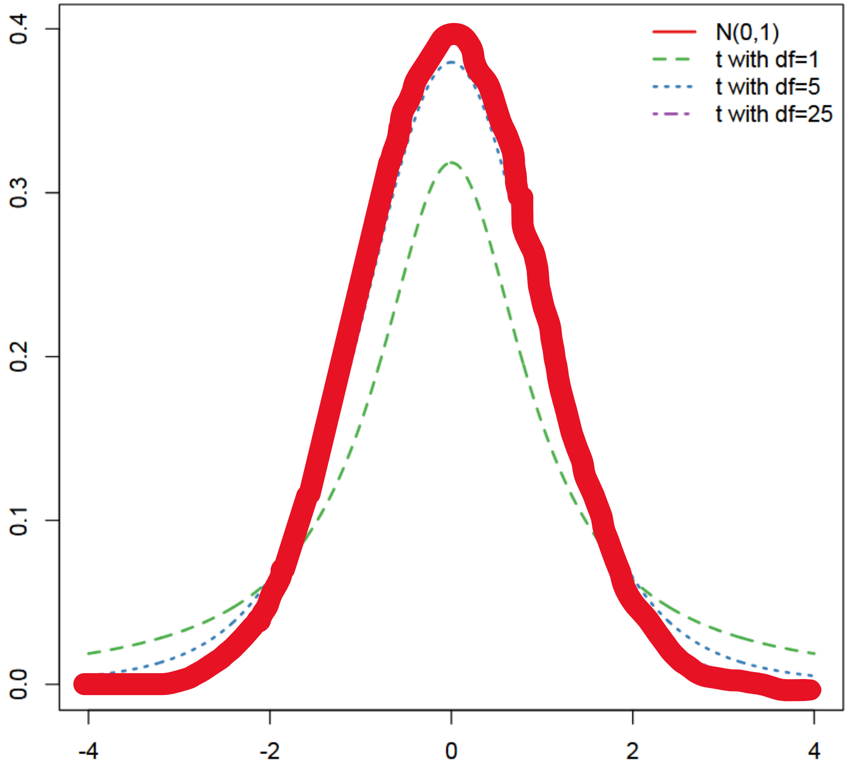

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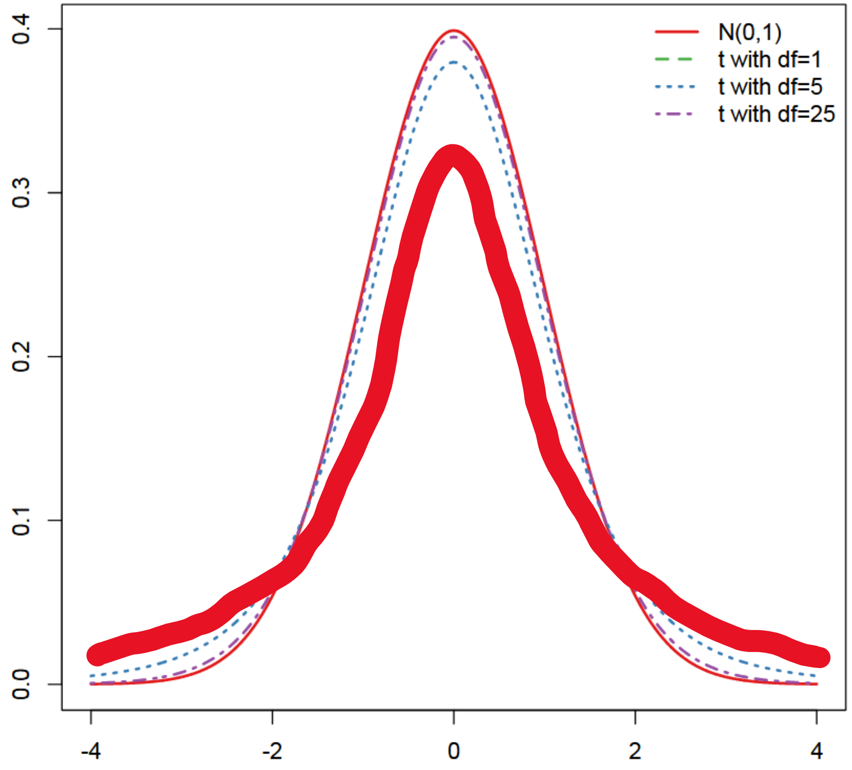

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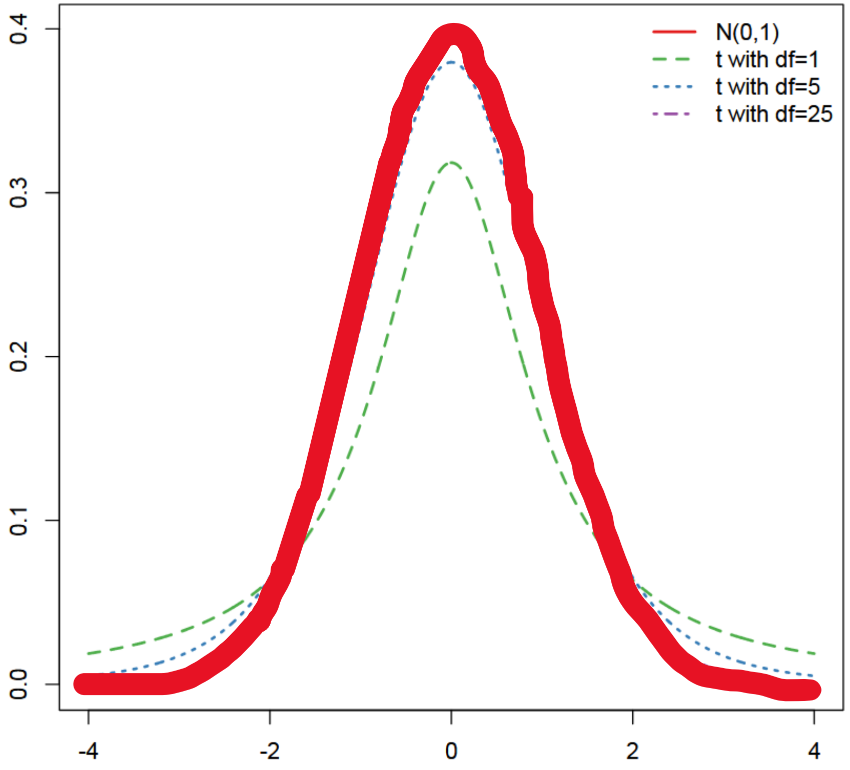

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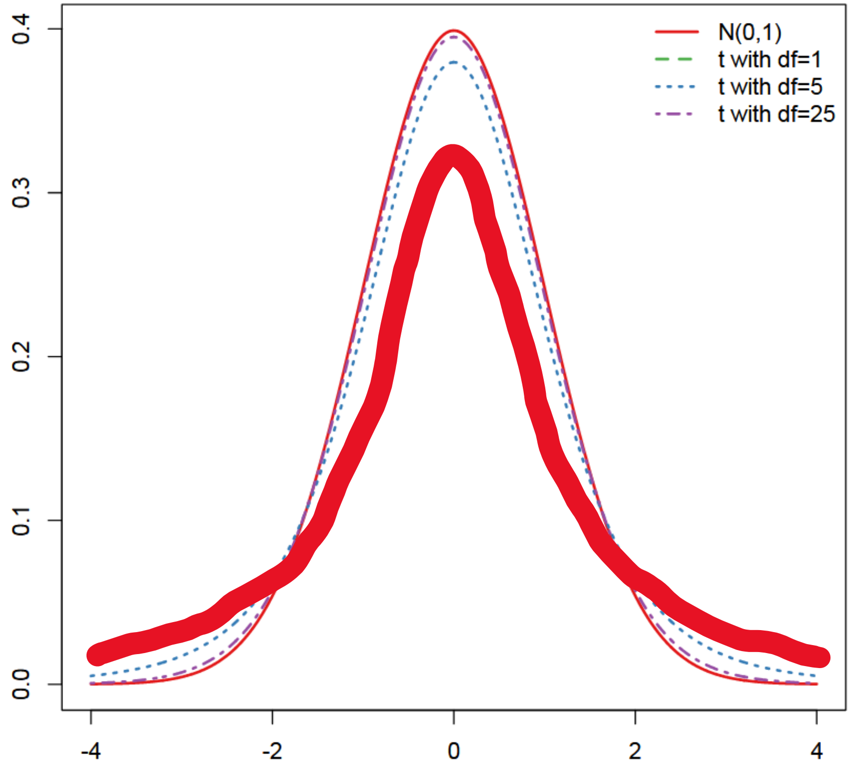

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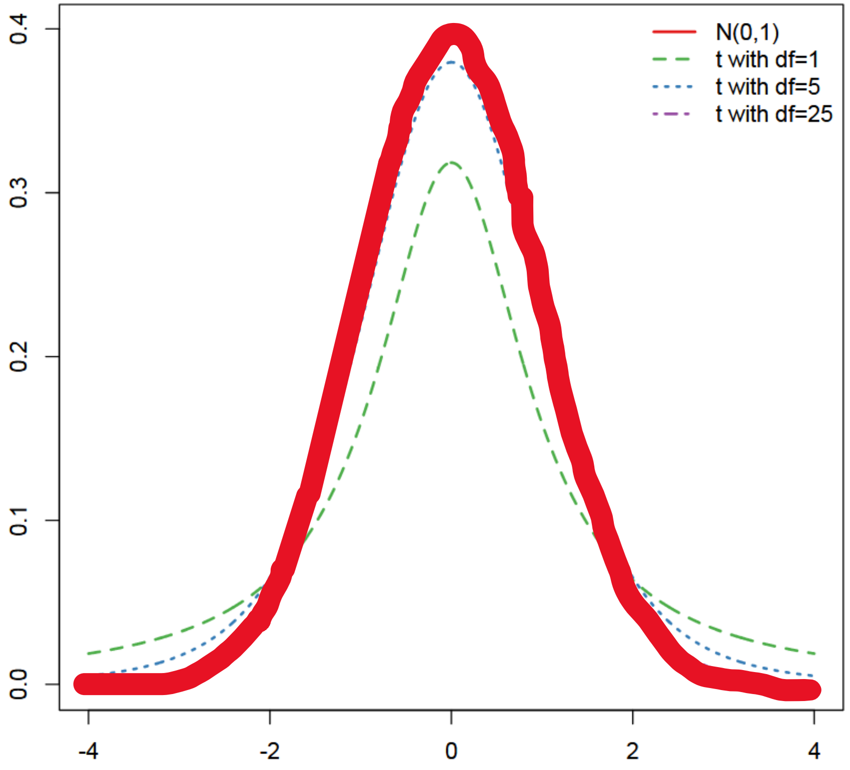

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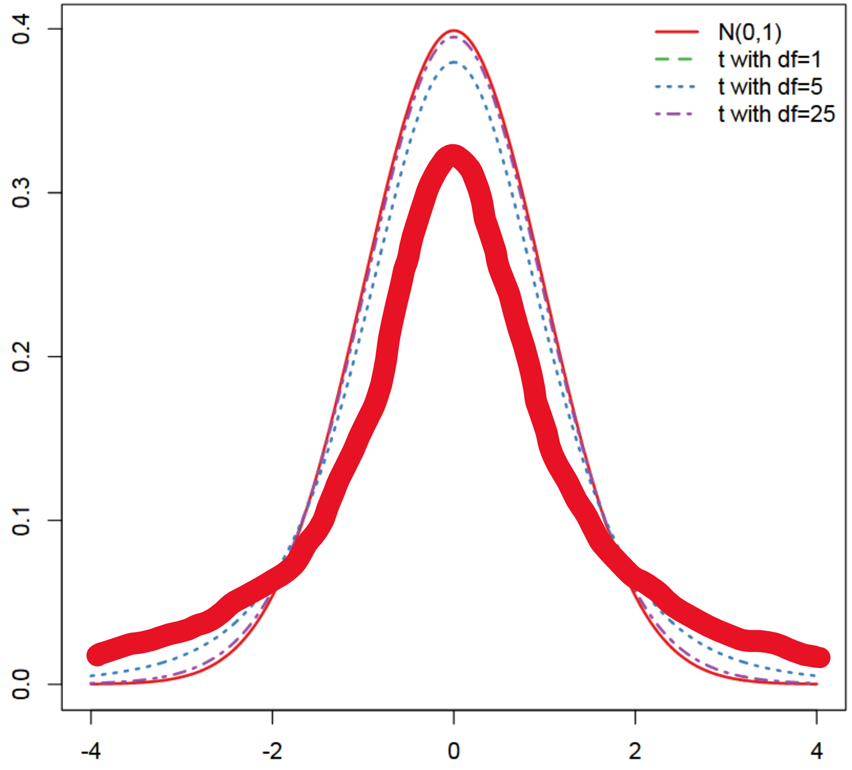
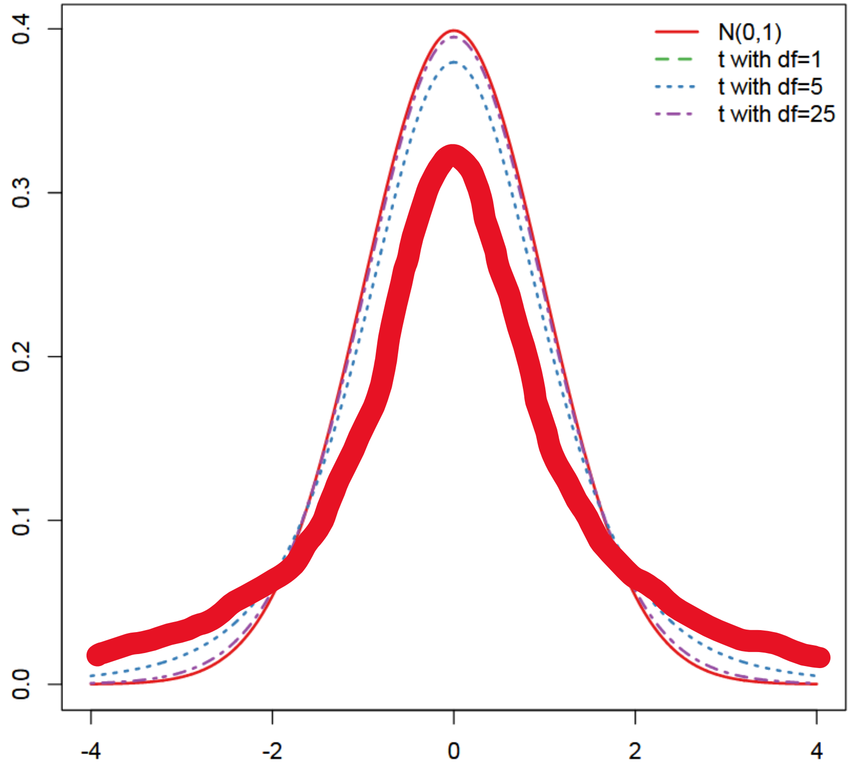


Sampling distribution of $t = \frac{\bar{x}-\mu}{s_x/\sqrt{n}}$ is a t_{n-1}-distribution. The $df = n-1$ is the **degrees of freedom (df)** of the t_{n-1}-distribution.

t_{n-1}-Distribution

Student's t_{n-1}-distribution is a family of symmetric curves fatter than the normal curve.

As sample size n increases, it becomes more like the normal curve.



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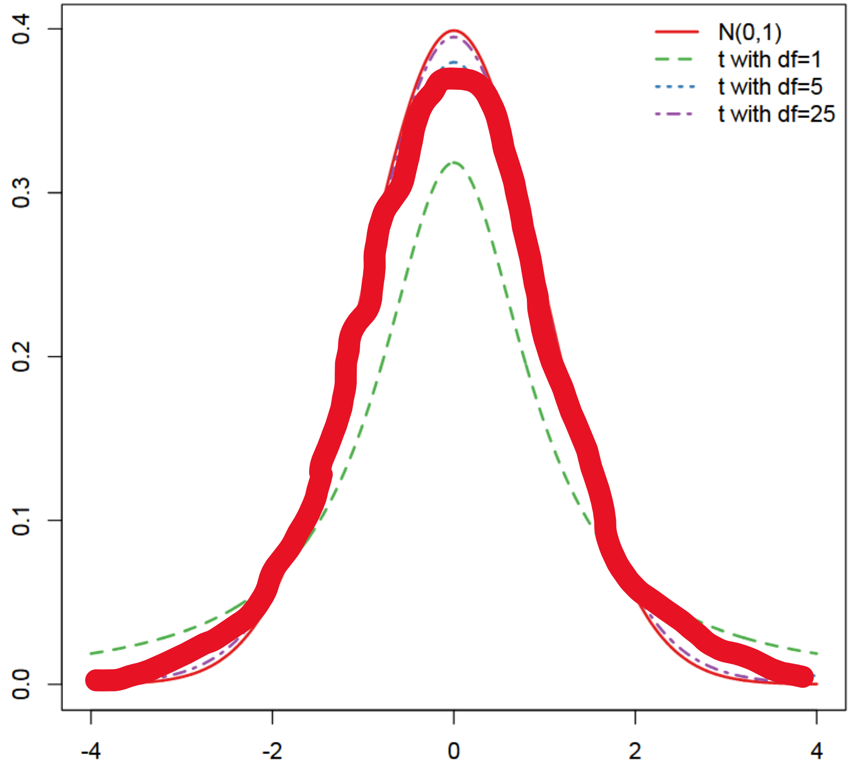

t_5-distribution

t_{n-1}-Distribution

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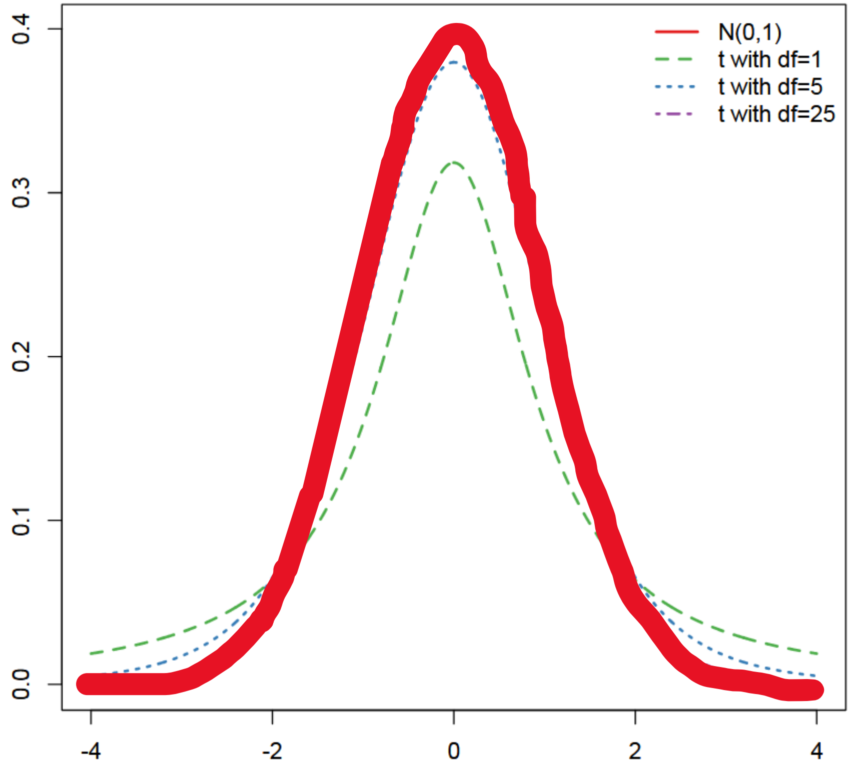

t_{25}-distribution

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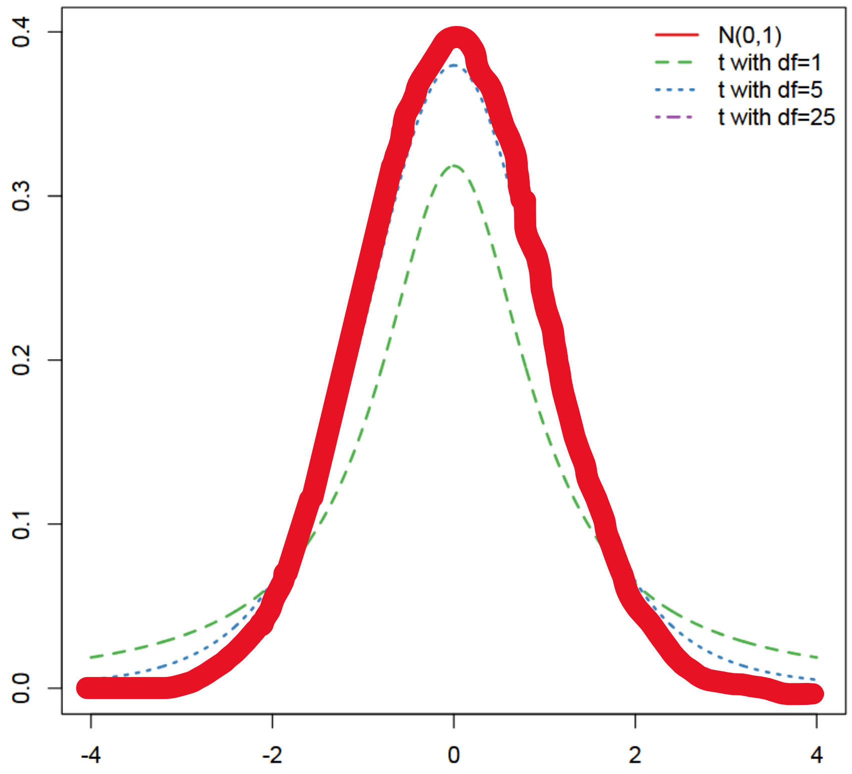

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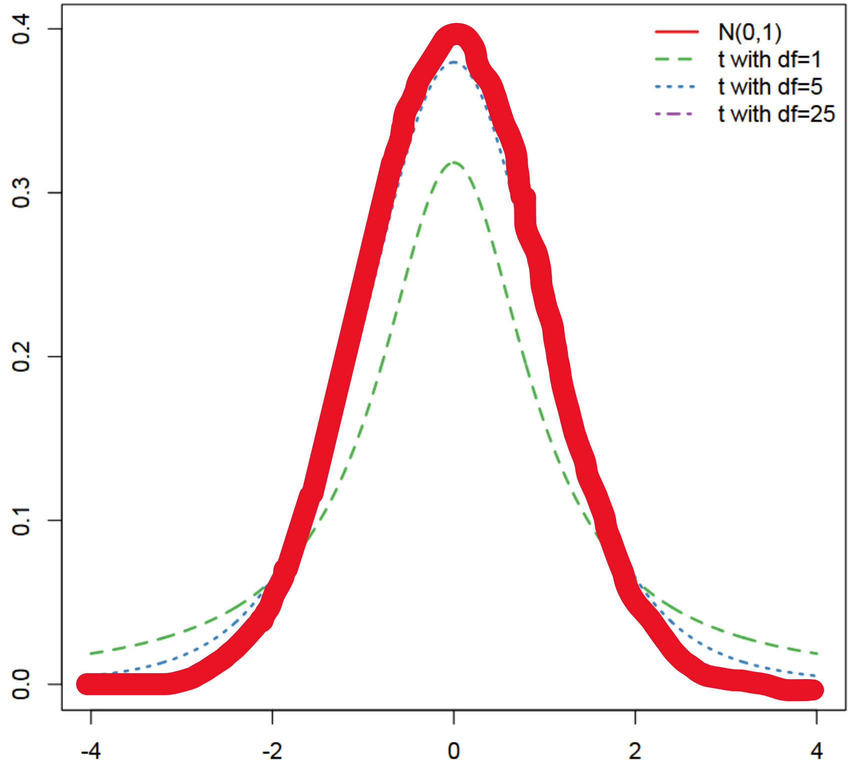

t_{25}-distribution

Confidence Interval of Mean with σ unknown (Facts)

The $\boldsymbol{c}$**-confidence interval for population mean** $\boldsymbol{\mu}$ obtained from an SRS with sample mean $\bar{x}$ and sample sd s_x is

$$\bar{x} - E < \mu < \bar{x} + E$$

with **margin of error**

$$E = t_{c,n-1} \frac{s_x}{\sqrt{n}}$$

when the following conditions are met:

- **10% Condition:** $N \geq 10n$.
- **Approx. normal:** Pop. distribution is normal or $n \geq 30$.

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Example 1.
Construct a 95% confidence interval for mean NOX exhaust level in all US cars given a sample of 40 readings with mean 1.268 and sample sd 0.333.

Confidence Interval of Mean with σ unknown (Example 1)

The **c-confidence interval for population mean μ** obtained from an SRS with sample mean $\bar{x}$ and sample sd s_x is

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Construct a 95% confidence interval for mean NOX exhaust level in all US cars given a sample of 40 readings with mean 1.268 and sample sd 0.333.

Answer.
$N \geq 10 \cdot 40$ and $n \geq 30$ so we can proceed.

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The $\boldsymbol{c}$**-confidence interval for population mean** $\boldsymbol{\mu}$ obtained from an SRS with sample mean $\bar{x}$ and sample sd s_x is

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Conclusion: We are 95% confident that the true mean NOX exhaust level is between 1.161 and 1.375.